Acceleration time scale for the first-order Fermi acceleration in relativistic shock waves.

J. Bednarz[1] & M. Ostrowski[1,2]

[1] *Obserwatorium Astronomiczne, Uniwersytet Jagielloński, Kraków, Poland (E-mail: bednarz@oa.uj.edu.pl and mio@oa.uj.edu.pl)*
[2] *Max-Planck-Institut für Radiostronomie, Bonn, Germany*

ABSTRACT
The acceleration time scale for the process of first-order Fermi acceleration in relativistic shock waves with oblique magnetic field configurations is investigated by the method of Monte Carlo particle simulations. We discuss the differences in derivation of the cosmic ray acceleration time scale for non-relativistic and relativistic shocks. We demonstrate the presence of correlation between the particle energy gain at interaction with the shock and the respective time elapsed since the previous interaction. Because of that any derivation of the acceleration time scale can not use the distribution of energy gains and the distribution of times separately. The time scale discussed in the present paper, $T_{acc}^{(c)}$, is the one describing the rate of change of the particle spectrum cut-off energy in the time dependent evolution. It is derived using a simplified method involving small amplitude particle momentum scattering and intended to model the situations with anisotropic cosmic ray distributions. We consider shocks with parallel, as well as oblique, sub- and super-luminal magnetic field configurations with finite amplitude perturbations, δB. At parallel shocks $T_{acc}^{(c)}$ diminishes with the growing perturbation amplitude and shock velocity U_1. Another feature discovered in oblique shocks are non-monotonic changes of $T_{acc}^{(c)}$ with δB. The effect arises due to the particle cross-field diffusion. The acceleration process leading to power-law spectra is possible in super-luminal shocks only in the presence of large amplitude turbulence. Then, $T_{acc}^{(c)}$ always *increases* with increasing δB. In some of the considered shocks the acceleration time scale can be shorter than the particle gyroperiod upstream the shock. We also indicate the relation existing for relativistic shocks between the acceleration time scale and the particle spectral index. A short discussion of the numerical approach modelling the pitch angle diffusion versus the large angle momentum scattering is given. We stress the importance of the proper evaluation of the effective magnetic field (including the perturbed component) in simulations involving discrete particle momentum scattering.

Key words: cosmic rays – shock waves – acceleration time scale – Fermi acceleration

1 INTRODUCTION

A consistent method to tackle the problem of first-order Fermi acceleration in relativistic shock waves was conceived by Kirk & Schneider (1987a; see also Kirk 1988). By extending the diffusion approximation to higher order terms in the anisotropy of particle distribution, they obtained solutions to a kinetic equation of the Fokker–Planck type with the isotropic form of pitch angle diffusion coefficient. Next, Kirk & Schneider (1988) extended the analysis by involving both diffusion and large-angle scattering in particle pitch angle. They discovered that – in relativistic shock waves – the presence of scattering can substantially modify the spectrum of accelerated particles. An extension of Kirk & Schneider's

(1987a) approach to more general conditions in the shock was given by Heavens & Drury (1988), who took into consideration the fluid dynamics of relativistic shock waves. They also noted that the resulting particle spectral indices depend on the perturbations spectrum near the shock, in contrast to the non-relativistic case. Kirk & Heavens (1989) considered the acceleration process in shocks with magnetic fields oblique to the shock normal (see also Ballard & Heavens 1991). They showed, contrary to the non-relativistic results again, that such shocks led to flatter spectra than the parallel ones. Their work relied on the assumption of adiabatic invariant $p_\perp^2/B$ conservation for particles interacting with the shock, which restricted the considerations to the case of nearly uniform magnetic fields upstream and downstream of

the shock. A different approach to particle acceleration was presented by Begelman & Kirk (1990), who noted that in relativistic shocks most field configurations lead to super-luminal conditions for the acceleration process. In such conditions, particles are accelerated in a single shock transmission by drifting parallel to the electric field present in the shock. Begelman & Kirk showed that there is more efficient acceleration in relativistic conditions than that predicted by the simple adiabatic theory. The acceleration process in the presence of finite amplitude perturbations of the magnetic field was considered by Ostrowski (1991; 1993) and Ballard & Heavens (1992). Ostrowski considered a particle acceleration process in the relativistic shocks with oblique magnetic fields in the presence of field perturbations, where the assumption $p_\perp^2/B = const$ was no longer valid. To derive particle spectral indices he used a method of particle Monte Carlo simulations and noted that the spectral index was not a monotonic function of the perturbation amplitude, enabling the occurrence of steeper spectra than those for the limits of small and large perturbations. It was also revealed that the conditions leading to very flat spectra involve an energetic particle density jump at the shock. The acceleration process in the case of a perpendicular shock shows a transition between the compressive acceleration described by Begelman & Kirk (1990) and, for larger perturbations, the regime allowing for formation of the wide range power-law spectrum. The analogous simulations by Ballard & Heavens (1992) for highly disordered background magnetic fields show systematically steeper spectra in comparison to the above results, as discussed by Ostrowski (1993). The particle spectrum formation in the presence of non-linear coupling of accelerated particles to the plasma flow has been commented by Ostrowski (1994a). The review of the above work is presented by Ostrowski (1994b).

The shock waves propagating with relativistic velocities rise also interesting questions concerning to the cosmic ray acceleration time scale, T_{acc}. To date, however, there is only somewhat superficial information available on that problem. A simple comparison to the non-relativistic formula based on numerical simulations shows that T_{acc} relatively decreases with increasing shock velocity for parallel (Quenby & Lieu 1989; Ellison et al. 1990) and oblique (Takahara & Terasawa 1990; Newman et al. 1992; Drolias & Quenby 1994; Lieu et al. 1994; Quenby & Drolias 1995; Naito & Takahara 1995) shocks. However, the numerical approaches used there, based on assuming the particle isotropization at each scattering, neglect or underestimate a rather significant factor controlling the acceleration process – the particle anisotropy. Ellison et al. (1990) and Naito & Takahara (1995) included also derivations applying the pitch angle diffusion approach. The calculations of Ellison et al. for parallel shocks show similar results to the ones they obtained with large amplitude scattering. In the shock with velocity $0.98\,c$ the acceleration time scale is reduced on the factor ~ 3 with respect of the non-relativistic formula. Naito & Takahara considered shocks with oblique magnetic fields. They confirmed reduction of the acceleration time scale with increasing inclination of the magnetic field, derived earlier for non-relativistic shocks (Ostrowski 1988). However their approach neglected effects of particle cross field diffusion and assumed the adiabatic invariant conservation at particle interactions with the shock. These two simplifications limit their results to

the cases with small amplitude turbulence near the shock[⋆]. One should also note that comparing in some of the mentioned papers the derived time scales to the non-relativistic expression does not have any clear physical meaning when dealing with relativistic shocks.

In order to consider the role of particle anisotropic distributions and different configurations of the magnetic field in shocks the present work is based on the small angle particle momentum scattering approach described by Ostrowski (1991). It enables to model the effects of cross-field diffusion, important in shocks with oblique magnetic fields. Let us note (cf. Ostrowski 1993) that this code allows for a reasonable description of particle transport in the presence of large amplitude magnetic field perturbations also. In Section 2 below, we discuss the differences in derivation of the cosmic ray acceleration time scale for non-relativistic and relativistic shock waves. We demonstrate the existence of noticeable correlations of the particle energy gains at interactions with the shock and the respective times elapsed since the previous interaction. Because of that any derivation of the acceleration time scale cannot use the distribution of energy gains and the distribution of times separately. We define the acceleration time scale $T_{acc}^{(c)}$, as the one describing the rate of change of the cut-off energy in the time dependent particle spectrum evolution. Then, in Section 3, the performed simulations are described. We use the simplified method involving small amplitude particle momentum scattering and devoted to model the situations with anisotropic cosmic ray distributions (Kirk & Schneider 1987b, Ellison et al. 1990, Ostrowski 1991). In Section 4 the results are presented for shock waves with parallel and oblique (both, sub- and super-luminal) magnetic field configurations. We consider field perturbations with amplitudes ranging from very small ones up to $\delta B \sim B$. In parallel shocks $T_{acc}^{(c)}$ diminishes with the growing perturbation amplitude and the shock velocity U_1. However, it is approximately constant at a given value of U_1 if we use the formal diffusive time scale as the time unit. Another qualitative feature discovered in oblique shocks is that due to the cross-field diffusion $T_{acc}^{(c)}$ can change with δB in a non-monotonic way. The acceleration process leading to the power-law spectrum is possible in super-luminal shocks only in the presence of large amplitude turbulence. Then, in contrast to the quasi-parallel shocks, $T_{acc}^{(c)}$ increases with the increasing δB. In some cases with the oblique magnetic field configuration one may note a possibility to have an extremely short acceleration time scales, comparable or even smaller than the particle gyroperiod in the magnetic field upstream the shock. We also demonstrate the existence of the coupling between the acceleration time scale and the particle spectral index. A form of the involved relation much depends on the magnetic field configuration. In order to evaluate some earlier simulations applying the large angle scattering ('LAS') model and/or the pitch angle diffusion ('PAD') model a short discussion of the results obtained within these two approaches is presented. We stress

---

[⋆] One should note that the spatial distributions near the shock derived by these authors (their figures 1 and 2) do not show the particle density jump proved to exist in oblique relativistic shocks by Ostrowski (1991). It is also implcitly present in analytic derivations of Kirk & Heavens (1989).

the importance of the proper evaluation of the effective magnetic field (including the perturbed component) in simulations involving discrete particle momentum scattering. In the final Section 5 we provide a short summary and discuss some astrophysical consequences of our results. Some preliminary results of this work were presented in Ostrowski & Bednarz (1995).

Below, the light velocity is used as the velocity unit, $c = 1$. As the considered particles are ultrarelativistic ones, $p = E$, we often put the particle momentum for its energy. In the shock we label all upstream (downstream) quantities with the index '1' ('2'). If not otherwise indicated, the quantities are given in their respective plasma rest frames. The shock *normal* rest frame is the one with the plasma velocity normal to the shock, both upstream and downstream the shock (cf. Begelman & Kirk 1990). In the present paper the acceleration time scales are always given in this particular frame.

2 THE ACCELERATION TIME SCALES IN NON-RELATIVISTIC VERSUS RELATIVISTIC SHOCK WAVES

In the case of non-relativistic shock wave, with velocity $U_1 \ll 1$, the acceleration time scale can be defined as

$$T_{acc} \equiv \frac{E}{\frac{\overline{\Delta E}}{\Delta t}} \quad ,$$

where $\overline{\Delta E}$ is the mean energy gain at particle interaction with the shock and Δt is the mean time between successive interactions. One can use mean values here because any substantial increase of particle momentum requires a large number of shock-particle interactions and the successive interactions are only very weakly correlated with each other. The respective expression for T_{acc} in parallel shocks,

$$T_{acc}^0 = \frac{3}{U_1 - U_2} \left\{ \frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2} \right\} \quad , \quad (2.1)$$

where κ_i is the respective particle spatial diffusion coefficient, has been discussed by Lagage & Cesarsky (1983). Ostrowski (1988) provided the analogous scale for shocks with oblique magnetic fields and small amplitude magnetic field perturbations. It can be written in the form

$$T_{acc}^\psi = \frac{3}{U_1 - U_2} \left\{ \frac{\kappa_{n,1}}{U_1 \sqrt{\frac{\kappa_{n,1}}{\kappa_{\parallel,1} \cos^2 \psi_1}}} + \frac{\kappa_{n,2}}{U_2 \sqrt{\frac{\kappa_{n,2}}{\kappa_{\parallel,2} \cos^2 \psi_2}}} \right\} \quad , \quad (2.2)$$

where the index n denotes quantities normal to the shock, the index $\parallel$ those parallel to the magnetic field, ψ is an angle between the magnetic field and the shock normal and $U_1/\cos \psi_1 \ll c$ is assumed. The terms $\sqrt{\kappa_n/(\kappa_\parallel \cos^2 \psi)}$ represent a ratio of the mean normal velocity of a particle to such velocity in the absence of cross-field diffusion. One may note that for negligible cross-field diffusion the expression (2.2) coincides with (2.1) if we put $\kappa_{n,i}$ for κ_i ($i = 1, 2$). The case of oblique shock with finite amplitude field perturbations has not been adequately discussed yet, but we expect the respective acceleration scale to be between the

values given by the above formulae for T_{acc}^0 and T_{acc}^ψ. The influence of the particle escape boundary on the acceleration time scale and the particle spectrum is discussed by Ostrowski & Schlickeiser (1996).

If the shock velocity becomes relativistic, the particle energy change at a single interaction with the shock can be comparable, or even larger than the original energy. Moreover, after interaction with the shock, the *upstream* particles with small initial angles between its momenta and the mean magnetic field have a larger chance to travel far away from the shock. On average, such particles spend longer times and are able to change its pitch angles substantially until the next hits at the shock. Then, larger pitch angles allow for particle reflections with large energy gains or transmissions downstream (cf. Ostrowski 1994b, also Ostrowski 1991, Lucek & Bell 1994). Therefore, correlations of the times between successive interactions, Δt_{diff}, the energy gains at these interactions, ΔE, and, possibly, the probability of particle escape occur. As an example, in Fig. 1 we map the number of particle interaction with the shock in coordinates $(\Delta t_{diff}, \Delta E)$ (a more detailed discussion of the existing correlations will be presented separately (Bednarz & Ostrowski, in preparation)). A cut of the presented surface at any particular value of Δt_{diff} gives the distribution of energy gains for particles who have spent this time since the last interaction with the shock. A general trend seen on the map for increasing Δt_{diff} is growing the value of $\Delta E/E$ for the distribution maximum. Because of that, we propose a different approach to derivation of the acceleration time scale with respect to that used for non-relativistic shocks. Usually the acceleration time scale is applied for derivation of the highest energies occurring in the particle spectrum, characterized by its cut-off energy, E_c. Thus we use this energy scale to define the acceleration time scale as:

$$T_{acc}^{(c)} \equiv \frac{E_c}{\dot{E}_c} \quad , \quad (2.3)$$

where $\dot{E}_c \equiv dE_c/dt$. The rate of the cut-off energy increase is a well-defined quantity and the time scale (2.3) has a clear physical interpretation. The above definition does not require any limit for the energy gains of the individual particles and all possible correlations are automatically included here. From the meaning of the definition (2.3) it follows that $T_{acc}^{(c)}$ is somewhat shorter than the respective scale at the same energy for later times, required for the respective part of the spectrum to become a pure power-law (cf. Ostrowski & Schlickeiser 1996). One should also note that in relativistic shocks the time scale depends on the reference frame we use for its measurement. In the present paper the acceleration time scales are given in the respective normal shock rest frame. However, the applied time units $r_{e,1}/c$ (see below) are defined with the use of the upstream gyration time.

3 SIMULATIONS

We derive estimates of $T_{acc}^{(c)}$ basing on the Monte Carlo particle simulations involving the small amplitude momentum scattering model of Ostrowski (1991) and 2500 particles in each run of the code. In the present considerations we discuss the role of the mean magnetic field configuration and
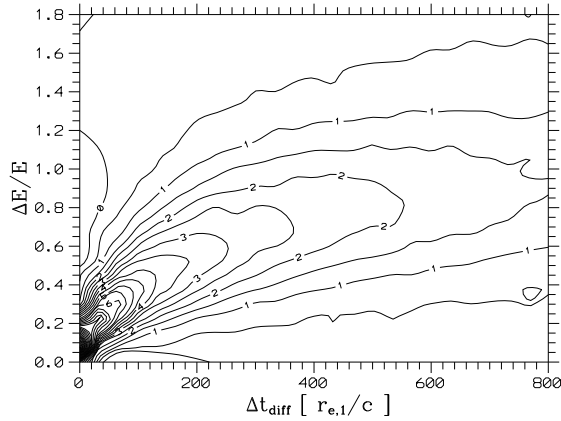
[hbt]

Figure 1. Distribution of particle-shock interaction events for upstream particles, $N(\Delta t_{diff}, \Delta E/E)$, in function of the upstream diffusive times, Δt_{diff}, and the respective energy changes, $\Delta E/E$. The correlation between Δt_{diff} and $\Delta E/E$ is represented by a regular drift of the distribution maximum toward higher $\Delta E/E$ when increasing Δt_{diff}. An example for the parallel shock with $U_1 = 0.5$, $\psi_1 = 1°$ and weak scattering conditions $(\kappa_\perp/\kappa_\parallel = 1.6 \cdot 10^{-6})$ is presented.

the amount of particle scattering. In order to avoid the additional effects of the varying shock compression due to the presence of different magnetic field configurations we take the field as a trace one, without any dynamical effects on the plasma flow. The shock compression, as seen in the normal shock rest frame, r, is derived from the approximate formulae presented by Heavens & Drury (1989). For illustration of the results, in the present paper we consider the shock waves propagating in the cold electron-proton plasma. For the *mean* magnetic field B_1 taken in the upstream plasma rest frame and inclined at the angle ψ_1 with respect to the shock normal we derive its downstream value and inclination, B_2 and ψ_2, with the use of jump conditions presented for relativistic shocks by e.g. Appl & Camenzind (1988):

$$B_2 = B_1 \sqrt{\cos^2\psi_1 + R^2 \sin^2\psi_1} \qquad , \qquad (3.1)$$

$$\tan\psi_2 = R \tan\psi_1 \qquad , \qquad (3.2)$$

where $R = r\,\gamma_1/\gamma_2$ and the Lorentz factors $\gamma_i \equiv 1/\sqrt{1 - U_i^2}$ $(i = 1, 2)$. These formulae are valid for both sub- and superluminal magnetic field configurations.

We model particle trajectory perturbations by introducing small-angle random momentum scattering along the mean-field trajectory. The particle momentum scattering distribution is uniform within a cone wide at $\Delta\Omega$ $(<< 1)$, along the original momentum direction. The present simulations use a constant value of $\Delta\Omega = 0.173$ $(= 10°)$. Scattering events are at discrete instants, equally spaced in time as measured in the units of the respective $r_{g,i}/c$ $(i = 1, 2)$. Here we affix a gyroradius with the index 'g' when it is a value given for the local *uniform* magnetic field component. Index 'e' means the *effective* field including the field pertur-

bations (see below). The increasing perturbation amplitude is reproduced in simulations by decreasing the time period Δt between the successive scatterings. For simplicity, we use the same scattering pattern ($\Delta\Omega$ and Δt in units of r_g/c) upstream and downstream the shock, leading to the same values of $\kappa_\perp/\kappa_\parallel$ in these regions (see, however, Ostrowski 1993). One should note that the particle momentum scattering due to presence of turbulent magnetic field is equivalent to the effective magnetic field larger than the respective uniform mean component, B_1 or B_2. In our model, the effective field can be estimated as

$$B_{e,i} = B_i \sqrt{1 + \left(0.67\frac{\Delta\Omega}{\Delta t}\right)^2} \quad (i = 1, 2) \qquad . \qquad (3.3)$$

It is the lower limit for actual field since the amount of power in perturbations with wave-lengths smaller than $c\,\Delta t$ cannot be considered within such a simple model. Below, the derived acceleration time scales are presented in units of the formal diffusive scale $T_0 \equiv 4(\kappa_{n,1}/U_1 + \kappa_{n,2}/U_2)/c$ or in units of $r_{e,1}/c$, in the shock normal rest frame.

Our numerical calculations involve particles with momenta systematically increasing over several orders of magnitude. In order to avoid any energy dependent systematic effect we consider the situation with all spatial and time scales – defined by the diffusion coefficient, the mean time between scatterings and the shock velocity – proportional to the particle gyro-radius, $r_g = p/(eB)$, i.e. to its momentum.

For a chosen shock velocity and the magnetic field configuration we inject particles in the shock at some initial momentum p_0 and follow their phase-space trajectories. We assume the constant particle injection to continue in time after the initial time $t_0 = 0$. As some particles escape from the acceleration process by crossing the escape boundary placed far behind the shock we use the trajectory splitting procedure to keep the total amount of particles involved in simulations constant (cf. Kirk & Schneider 1987b; Ostrowski 1991). Here we put the boundary at the distance $6\kappa_{2,n}/U_2 + 4r_{g,2}$. We checked by simulations that any further increase of this distance does not influence the results in any noticeable way. For every shock crossing, the particle weight factor multiplied by the inverse of the particle velocity normal to the shock ($\equiv$ particle density) is added to the respective time and momentum bin of the spectrum, as measured in the shock normal rest frame. As one considers a continuous injection in all instants after t_0, in order to obtain the particle spectrum at some time $t_j > t_0$, one has to add to particle density in a bin p_i at t_j the densities in this momentum bin for all the earlier times. The resulting particle spectra are represented as power-law functions with the squared exponential cut-off in momentum

$$f(p,t) = A\,p^{-\alpha}\,e^{-\left(\frac{p}{p_c}\right)^2} \qquad . \qquad (3.4)$$

In this formula three parameters are to be fitted: the normalization constant A, the spectral index for the stationary solution α, and the momentum cut-off p_c (Fig. 2; for details of the fitting procedure see Appendix A).

In the simulations, due to our, proportional to momentum, scaling of the respective quantities, the derived acceleration time scale (2.3) must be also proportional to p, and thus to $r_g(p)/c$. Therefore, this time scale measured in units
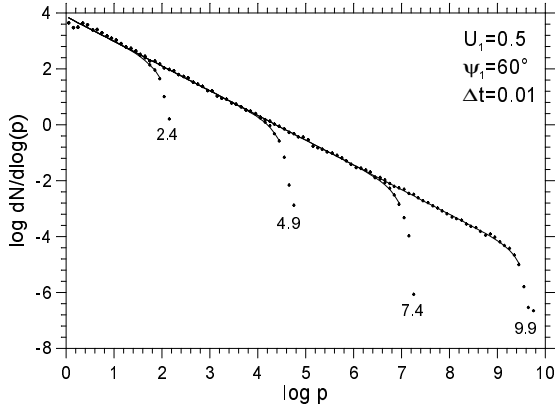
[hbt]

Figure 2. Particle spectra at a sequence of time instants. Individual simulation points represent the particle number (weight) ΔN per logarithmic momentum bin $\Delta \log p$. With full lines we present the respective fits (3.4).

of $r_g(p_c)/c$ (or $r_e(p_c)/c$) is momentum independent and can be easily scaled to any momentum. The parameter T_r gives the value of the acceleration time scale in units of $r_{e,1}/c$, $T_{acc}^{(c)} = T_r \, r_{e,1}/c$. The value of $T_{acc,i}^{(c)}$ at a particular time t_i is derived from the respective values of $p_{c,i}$:

$$T_{acc,i}^{(c)} = \frac{p_{c,i}}{\frac{p_{c,i}-p_{c,i-1}}{t_i-t_{i-1}}} \qquad , \qquad (3.5)$$

where we consider the advanced phase of acceleration ($p_{c,i} \gg p_0$). As in our simulations $p_c \propto t$ the condition $(p_{c,i} - p_{c,i-1})/p_{c,i} \ll 1$ is not required to hold in equation (3.5). Therefore, with all scales proportional to the particle momentum, the formula (3.5) reduces to $T_{acc,i}^{(c)} = t_i$ and the parameter T_r tends to a constant (Fig. 3). The extension of the simulated spectra over several decades in particle energy allows to avoid problems with the initial conditions and decrease the relative error of the derived time scale by averaging over a larger number of instantaneous $T_{acc,i}$.

4 THE ACCELERATION TIME SCALE IN RELATIVISTIC SHOCK WAVES

For a given relativistic shock velocity particle anisotropy in the shock depends on the mean magnetic field inclination to the shock normal and the form of turbulent field. Below, we describe the results of simulations performed in order to understand the time dependence of the acceleration process in various conditions. In order to do that, we consider shock waves propagating with velocities $U_1 = 0.3$, 0.5, 0.7 and 0.9 of the velocity of light, and the magnetic field inclinations including the quasi-parallel, oblique sub-luminal and oblique super-luminal configurations. In all these cases we investigate the role of varying magnitude of turbulence characterized here by the value of Δt or by the ratio of the diffusion coefficient across the mean field and that along the field, $\kappa_\perp/\kappa_\parallel$. The relation between these parameters for $\Delta\Omega = 10°$ is
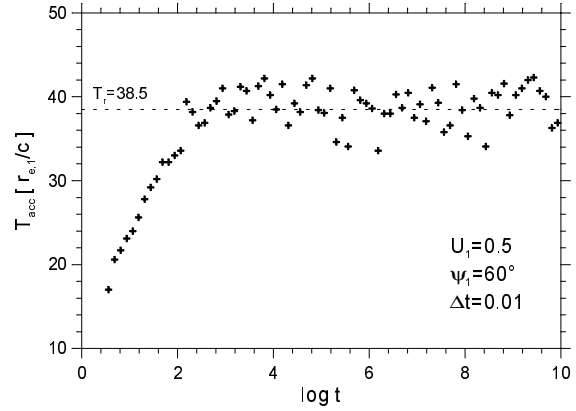
[hbt]

Figure 3. An example of simulated values of $T_{acc}^{(c)}$ in units of $r_{e,1}/c$ in a single run of the code. The simulation time t is given in units of $r_{g,1}(p = p_0)/c$ in the shock normal rest frame. We fit the final acceleration time scale only to the points at the advanced phase of acceleration (dashed line). The dispersion of these points defines the fitting error.

presented at Fig. 4, where - at $\Delta t > 0.01$ - the presented relation has the power-law form $\kappa_\perp/\kappa_\parallel = 6.3 \cdot 10^{-5} \, (\Delta t)^2$.

4.1 Parallel shocks

The most simple case for discussion of the first-order Fermi acceleration is a shock wave with parallel configuration of the mean magnetic field. As an example we consider the shock with negligible field inclination $\psi_1 = 1°$. For such a shock, the present simulations do confirm the expected relation of decreasing the acceleration time scale with increasing the shock velocity and the amplitude of trajectory perturbations (Fig. 5). One should note at the upper panel of the figure that for short Δt the presented time scales decrease more and more slowly with decreasing Δt . It is due to the fact that starting from some value of Δt we reach conditions of nearly isotropic diffusion, $\kappa_\parallel \approx \kappa_\perp$, and further decreasing of the time delay between scatterings decreases the acceleration time in much the same proportion as the time unit $r_{e,1}/c$ used to measure it (cf. Equ. 3.3, Fig. 4). In the lower panel of Fig. 5 the diffusive time scale T_0 is used as the time unit. The minute differences between the successive curves reflect the statistical fluctuations arising during simulations. Without such fluctuations all curves should coincide. The one sigma fit errors of $T_{acc}^{(c)}$ are indicated near the respective points. One should note that for increasing the shock velocity the acceleration time scale decreases with respect to the diffusive time scale.

4.2 Variation of $T_{acc}^{(c)}$ with magnetic field inclination

In order to compare the acceleration time scales for different magnetic field inclinations ψ_1 we performed simulations assuming a constant scattering parameters upstream and
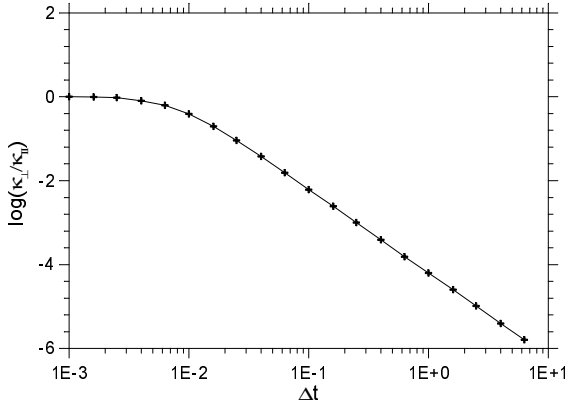
[hbt]

Figure 4. The relation between the scattering parameter Δt and the respective ratio of diffusion coefficients $\kappa_\perp/\kappa_\parallel$.
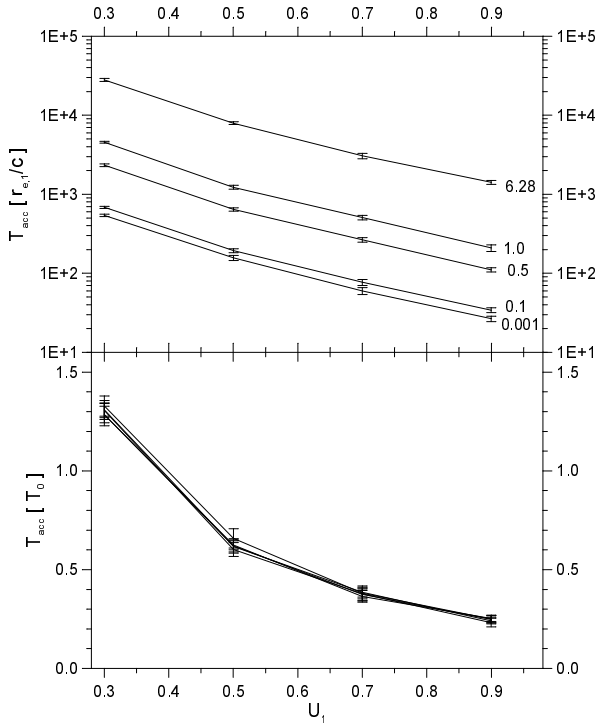
[hbt]

Figure 5. The values of $T_{acc}^{(c)}$ in parallel shock waves ($\psi_1 = 1°$) in units of a.) $r_{e,1}/c$ and b.) T_0 versus the shock velocity U_1. The values resulting from simulations are given for $U_1 = 0.3$, 0.5, 0.7 and 0.9 with the scattering amplitude parameter Δt near the respective results at the upper panel.

downstream, yielding the same ratio of $\kappa_\perp/\kappa_\parallel$ in these regions. However, due to shock compression the particle gyration period is shorter downstream than upstream, in proportion to the *mean* magnetic field compression (3.1). At Fig. 6 we present the values of the acceleration time scale derived in such conditions at different ψ_1. For super-luminal shocks the results are presented for the cases allowing for particle power-law energy spectra, i.e. when the cross-field diffusion is sufficiently effective. Actually, the spectra with inclinations $\alpha < 10.0$ are only included. We consider the following values of the magnetic field inclination: $\psi_1 = 1°$, $25.8°$, $45.6°$, $60°$, $72.5°$, $84.3°$ and $89°$. The first one is for a parallel shock, the last two ones are for perpendicular super-luminal shock with all velocities U_1. The intermediate values define luminal shocks ($U_1/\cos\psi_1 = 1.0$) at the successive velocities considered, respectively $U_1 = 0.9$, 0.7, 0.5 and 0.3.

In general, the acceleration time scale decreases with increasing field inclination, reaching in some cases the values comparable, or even smaller than the particle upstream gyroperiod (6.28 in our units of $r_{e,1}/c$). The trend can be reversed for intermediate wave amplitudes when the magnetic field configuration changes into the luminal and super-luminal one. Such changes are accompanied with the steepening of the spectrum (see below). The acceleration rate at different scattering amplitudes changes with ψ_1 in a way that at different inclinations the minimum acceleration times occur at different perturbation amplitudes (different Δt).

An important feature of the acceleration process in relativistic shocks should be mentioned at this point. The variations of $T_{acc}^{(c)}$ in oblique shocks are accompanied by changes of the particle spectrum inclination (cf. Kirk & Heavens 1989; Ostrowski 1991). At Fig. 7, the curves at ($T_{acc}^{(c)}$, α) plane represent the results for decreasing the scattering amplitude expressed with parameter Δt, and joined with lines for the same magnetic field inclination ψ_1. For parallel shocks the changes in $T_{acc}^{(c)}$ do not lead to any variation of the spectral index. However, for oblique sub-luminal ($\psi_1 = 25.8°$, $45.6°$) and luminal ($\psi_1 = 60°$) shocks a non-monotonic behaviour is seen. The trend in changing $T_{acc}^{(c)}$ and α observed at smaller perturbation amplitudes (larger Δt) is reversed at larger amplitudes, when the substantial cross-field diffusion is possible.

4.3 Variation of $T_{acc}^{(c)}$ with varying turbulence level

In parallel shocks the acceleration time scale reduces with the increased turbulence level in its neighbourhood. This phenomenon, well known for non-relativistic shocks (cf. Lagage & Cesarsky 1983), is confirmed here for relativistic shock velocities (Fig. 5). In general, there are two main reasons for this change. The first one is a simple reduction of the diffusion time of particles outside the shock due to shorter intervals between scatterings, analogous to the decrease observed in non-relativistic shocks. However, the increased amount of scattering influences also the acceleration process due to changing (decreasing) the particle anisotropy at the shock and, thus, modifying the mean energy gain of particles interacting with the shock discontinuity. Additionally, in oblique shocks the upstream-downstream transmission probability may increase. One should note that the present approach is not able to describe fully the effect of de-
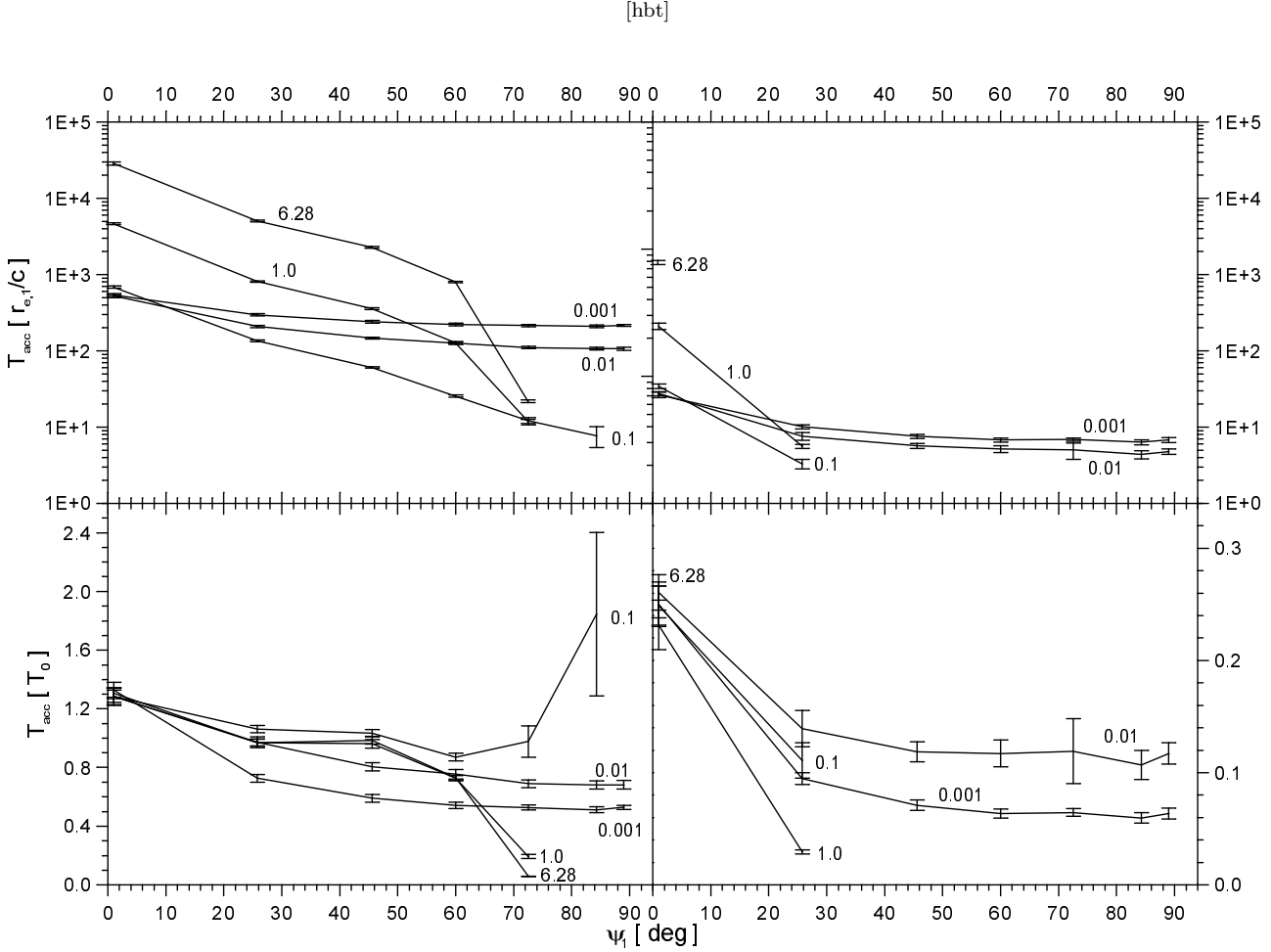
[hbt]

Figure 6. The value of $T_{acc}^{(c)}$ in units of $r_{e,1}/c$ (upper panels) and T_0 (lower panels) versus the magnetic field inclination ψ_1. The values resulting from simulations are presented for $U_1 = 0.3$ and 0.9, with values of the parameter Δt given near the respective results.

creasing anisotropy with the small amplitude random scattering model applied. It is due to the fact that correlations between the successive modifications of a trajectory (a sequence of small angle scattering acts in this paper) in a single MHD wave cannot be accurately modelled within the simplified approach used. A more exact approach requires integration along the particle trajectories in realistic configurations of the magnetic field. However, the comparison of the present simplified method to the one involving such an integration shows a reasonably good agreement (Ostrowski 1993) suggesting that averaging over realistic trajectories is equivalent in some way to such averaging within our random scattering approach.

In shocks with oblique magnetic fields a non-monotonic change of the acceleration time scale with the amount of scattering along the particle trajectory is observed (Fig. 8, see also Fig. 6; cf. Ostrowski 1991, 1994b for the spectral index). Increasing the amount of turbulence up to some critical amplitude decreases the diffusion time along the magnetic field and thus $T_{acc}^{(c)}$. However, as the mean diffusion time outside the shock is related to the normal diffusion coefficient[†] κ_n ($\kappa_{n,i} = \kappa_{\|,i} \cos^2 \psi_i + \kappa_{\perp,i} \sin^2 \psi_i$, $i = 1, 2$), the increasing $\kappa_\perp$ will lead – for large scattering amplitudes – to longer T_{acc} in units of $[r_e/c]$. In the units of T_0 the acceleration time depends only weakly on the turbulence level and shows a small maximum for the minimum at the presented figure. For super-luminal shocks one can note absence of data points corresponding to low turbulence levels, where the power-law spectrum cannot be formed or it is extremely steep. In these excluded cases, the upstream population of energetic particles is only compressed at the shock with the characteristic upstream time of $\sim r_{e,1}/U_1$ (cf. Begelman & Kirk 1990; Ostrowski 1993).

[†] One should note that for the relativistic shocks, due to particle anisotropy, the respective relation may be not so simple as that given in equation (2.2) for non-relativistic shocks.
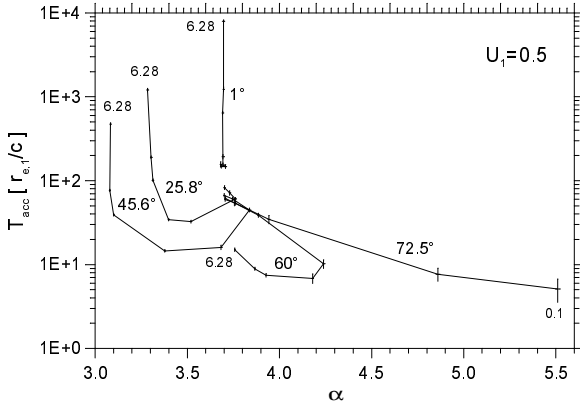
[hbt]

Figure 7. The values of $T_{acc}^{(c)}$ in units of $r_{e,1}/c$ at different inclinations ψ_1 versus the particle spectral index α. The values resulting from simulations are given for $U_1 = 0.5$ for five values of the angle ψ_1 given near the respective results. The *maximum* value of Δt is given at the end of each curve and it monotonously decreases along the curve.
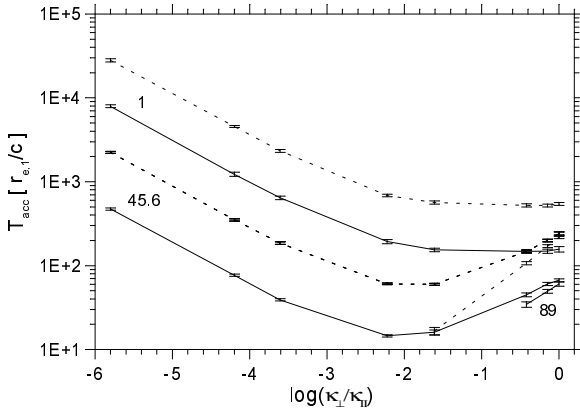
[hbt]

Figure 8. The acceleration time $T_{acc}^{(c)}$ versus the level of particle scattering $\kappa_\perp/\kappa_\parallel$ for shocks with velocity $U_1 = 0.3$ (dashed lines) and 0.5 (full lines). We present results for three values of the magnetic field inclination: a.) parallel shock ($\psi_1 = 1°$), b.) a sub-luminal shock with $\psi_1 = 45.6°$ and c.) a super-luminal shock with $\psi_1 = 89°$.

4.4 Large angle scattering (LAS) versus pitch angle diffusion (PAD)

Some earlier derivations of the acceleration time scale were based on the numerical simulations involving particle scattering at point like scattering centres isotropizing the particle momentum at each scattering. This approach does not provide a proper description for the acceleration processes in shock waves moving with velocities comparable to the particle velocity because it removes particle anisotropy and changes the factors related to it. Moreover, against argu-

ments presented in some papers, such scattering pattern can not be realized in turbulent magnetic fields near relativistic shocks, where most particles active in the acceleration process are able do diffuse only a short distance, below a few particle gyroradii off the shock[‡]. Such distances are often insufficient to allow for big particle pitch-angle changes occuring with the *point-like* scattering centres which isotropize particle momentum at each scattering. In shocks with oblique magnetic fields such scattering pattern can substantially change the shape of the accelerated particle spectrum with respect to the PAD model. Additionally, as an individual particle interaction with the shock can involve a few revolutions along the magnetic field, the usually assumed adiabatic invariant conservation, $p_\perp^2/B = const$, cannot be valid for short inter-scattering intervals.

Before proceeding with the results let us remark a further problem arising within any model involving discrete particle scattering acts. As discussed in Section 3 the presence of scattering is equivalent to the presence of magnetic field perturbations. As a result the effective field is larger than the uniform background field. The presented estimate of the this field (Equ. 3.3) is valid for small angle momentum scattering. The amount of energy in magnetic turbulence with the waves shorter than $c\,\Delta t$ is required to be small because the presented estimate assumes the particle momentum perturbation in Δt occurs on the uniform effective perturbing field. To compare the scattering processes with different Δt one have to neglect the unknown factor of the ratio of the averaged actual magnetic field to the estimated value (like the one in Equ. 3.3). Let us note that this factor, as well as the notion of the effective field were not considered in the earlier papers. In derivation of the results presented below we considered either the Equ. (3.3) to be formally valid for both, PAD and LAS scattering models, or we presented data with the use of units defined by the particle gyroradius in the uniform magnetic field component B_0. The later case is provided to be compared with the earlier results by other authors.

In order to have the comparison of the LAS versus PAD models meaningful we simulate the acceleration process for LAS with the described in Section 3 procedure, but applying the isotropic scattering of particle momentum. Instead of our $\Delta\Omega = 10°$ for the pitch angle diffusion we use $\Delta\Omega = 180°$ for the large angle scattering. The times Δt between successive scatterings are chosen in a way to have the same spatial diffusion coefficients along the magnetic field for both models, as measured in the units related to the *effective* field component (r_e and r_e/c). With the formulae derived in simulations ($\kappa_\parallel^{PAD} = 41.7\Delta t_{10°}$ for PAD and $\kappa_\parallel^{LAS} = 0.174\Delta t_{180°}$ for LAS) given in the units based on B_0 (r_g and r_g/c), and with the equation (3.3) for the effective field B_e one obtains the required relation of the scattering times: $\Delta t_{180°} = [57611\Delta t_{10°}^2 + 775.4]^{1/2}$. Let us note that as our procedure derives the actual particle trajectories (not only the gyration centre motion) any random scattering process results in the cross-field diffusion and breaking the strict $p_\perp^2/B$ conservation.

[‡] However, for the non-relativistic shock velocity and particles much above the injection energy such approximation can be safely used (cf. Jones & Ellison 1991).
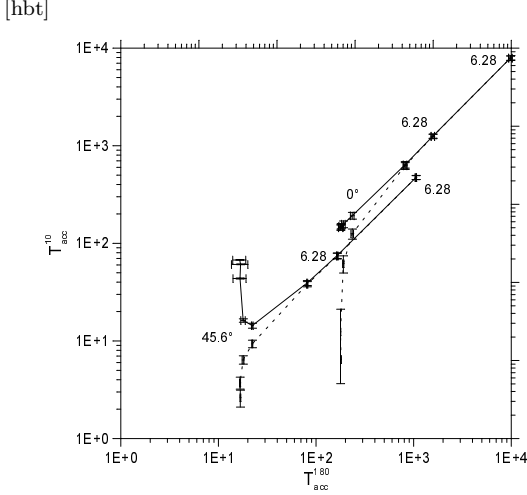
[hbt]

Figure 9. Comparison of the acceleration times derived with our pitch angle diffusion procedure, $T_{acc}^{10} \equiv T_{acc}^{(c)}(\Delta\Omega = 10°)$, and the large angle scattering model, $T_{acc}^{180} \equiv T_{acc}^{(c)}(\Delta\Omega = 180°)$. At the figure the points for the shock velocity $U_1 = 0.5$ and two magnetic field inclinations, $\Psi_1 = 0°$ and $45.6°$, are joined with lines for successive values of Δt. Full (dashed) lines are for the data in the units defined by B_e (B_0), respectively. The points for maximum Δt (= 6.28 in PAD) are indicated near the respective curves.
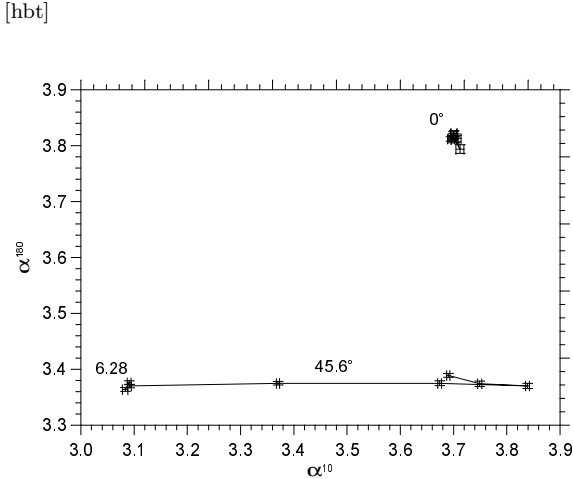
[hbt]

Figure 10. Comparison of the spectral indices derived with our pitch angle diffusion procedure, $\alpha^{10} \equiv \alpha(\Delta\Omega = 10°)$, and the large angle scattering model, $\alpha^{180} \equiv \alpha(\Delta\Omega = 180°)$. The data are presented for the shock wave with $U_1 = 0.5$ and the magnetic field inclinations $0°$ and $45.6°$. At the figure the data are plotted in the same way as at Fig. 9.

The discussion in this sub-section is to clarify the relation of our present simulations to some earlier ones providing sufficient details on the applied numerical approach. We performed simulations with the use of the LAS model for $U_1 = 0.3$, 0.5, 0.7 and 0.9, and two inclination angles, $\psi_1 = 0.1°$ and $45.6°$. The results for $U_1 = 0.5$ are presented at Fig-s 9 and 10. The comparison to the earlier results done for the largest Δt values, where the effective field approx-
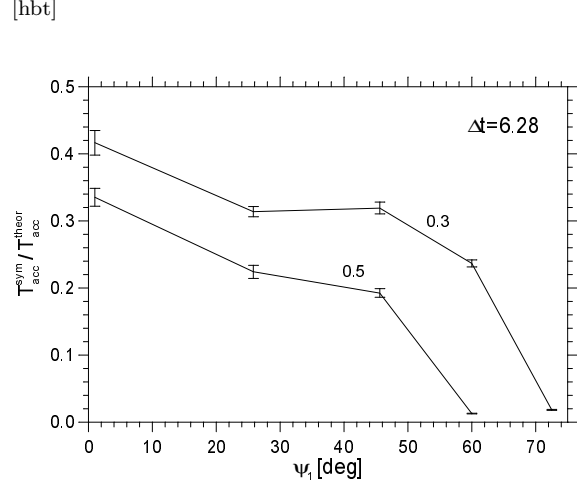
[hbt]

Figure 11. The ratio of the PAD simulated time scale T_{acc}^{sym} to the 'theoretical' non-relativistic time scale T_{acc}^{theor} given in Equ. 2.2 for different inclinations of the magnetic field Ψ_1. The results for $U_1 = 0.3$ and 0.5 are presented for the case of negligible cross-field diffusion ($\Delta t = 6.28$ in the PAD model).

imately equals the background field, shows a general qualitative agreement with the results of Ellison et al. (1990) and Naito & Takahara (1995). For parallel shocks the acceleration time scale for LAS is only slightly longer than the PAD one for all considered shock velocities. The ratio of this time to the time scale estimate based on the equation (2.1) for non-relativistic shocks yields values decreasing from approximately 0.6 at $U_1 = 0.3$ up to 0.17 at $U_1 = 0.9$. At high velocities the ratio is much smaller (a factor of $2 - 3$) than the ones derived by Ellison et al. (1990). However, there exist a substantial difference of numerical procedures applied here and in the mentioned paper, and we are not able to point out any single reason for the noted discrepancy. As mentioned previously our time scale defined with the use of the cut-off energy is shorter than the steady-state estimate derived by Ellison et al. Additionally, in the mentioned paper the use of analytic solutions of the spatial *diffusion* equation for the case of strongly relativistic flow can be questioned. For oblique shocks we observe analogous reduction of the acceleration time scale as that reported by Naito & Takahara (1995), with the PAD model allowing for a little more rapid acceleration than the LAS model. Of course this agreement is broken for short Δt, where the cross field diffusion can not be neglected and the particle magnetic momentum is not conserved at interactions with the shock. At Fig. 10 one can compare the spectral indices derived with the use of both approaches. As a final remark we would like to point out that the only independent check for any numerical procedure is provided by comparing of the derived spectral indices in the limiting cases of $\delta B << B$ and $\kappa_\perp \approx \kappa_\parallel$ to the analytic results (e.g. Heavens & Drury (1988) for parallel shocks and Kirk & Heavens (1989) for oblique shocks). This comparison for our PAD method is presented in Ostrowski (1991).

5   SUMMARY AND FINAL REMARKS

In the present paper we consider the acceleration time scale in relativistic shock waves. We argue that the numerical approach based on the model involving particle isotropic scattering neglects or underestimates a very significant factor controlling the acceleration process – the particle anisotropy. We also note that such a scattering pattern cannot be physically realized when particle trajectory perturbations are due to MHD waves. It is why the derivations of the present work are based on the small-angle momentum scattering approach. We note that the code allows for a satisfying description of particle transport in the presence of large amplitude magnetic field perturbations as well. We demonstrate the existence of correlation between particle energy gains and its diffusive times. The analogous correlation is expected for the probability of particle escape downstream the shock. Therefore, for defining the acceleration time scale we use the rate of change of the spectrum cut-off momentum which acommodate all such correlations. We performed Monte Carlo simulations for shock waves with parallel and oblique (either, sub-luminal and super-luminal) magnetic field configurations with different amounts of scattering along particle trajectories. Field perturbations with amplitudes ranging from very small ones up to $\delta B \sim B$ are considered. In parallel shocks $T_{acc}^{(c)}$ diminishes with the growing perturbation amplitude and the shock velocity. However, it is approximately constant for increasing turbulence level if we use the respective diffusive time scale as the time unit. Another feature discovered in oblique shocks is that due to the cross-field diffusion $T_{acc}^{(c)}$ can change with δB in a non-monotonic way. The acceleration process leading to the power-law particle spectrum in a super-luminal shock is possible only in the presence of large amplitude turbulence. Then, the shorter acceleration times occur when the perturbations' amplitudes are smaller and the respective spectra steeper. We discussed the coupling between the acceleration time scale and the particle spectral index in oblique shock waves with various field inclinations and revealed a possibility for non-monotonic relations of these quantities.

The shortest acceleration time scales seen in the simulations are below the particle gyroperiod upstream the shock. These times do not require the ultra-relativistic shock velocities, but may occur in mildly relativistic ones with the quasi-perpendicular magnetic field configuration. One should note that - due to the larger magnetic field downstream the shock - in this short time the particle trajectory can follow a few revolutions near the shock with only a short section of each one penetrating the upstream region.

The presented estimates of the acceleration time scale provide an interesting possibility for modelling shock waves in the conditions where the electron spectrum cut-off energy is determined by the balance of gains and losses. If one is able to derive the respective acceleration rate from the knowledge of the energy loss process, and the particle spectral index is also known, then both these values provide constraints for the acceleration process which could be further used to reduce the parameter space available for the considered shock wave (cf. Fig. 7).

APPENDIX A. FITTING THE PARAMETERS A, α AND P_C

Any simulated spectrum was evolving in time by increasing the width of its power-law section and, thus, the best fit of this power-law was possible with the use of the final spectrum at maximum time. Therefore, in the simulations we used the last spectrum to fit parameters of the power-law normalization and the spectral index, A and α. Next, for any earlier spectrum, these parameters were assumed to be constant and we were fitting only the cut-off momentum, p_c. For each fit we used 20 last points of the spectrum preceding the point where particle density fell below 0.16 of $A p^{-\alpha}$. The number of 0.16 was chosen experimentally as to obtain the best fits to the cut-off region of the spectrum. As the distribution (3.4) represents only an approximation to the actual particle distribution, there was no reason to use points corresponding to lower densities, of lesser statistical significance.

REFERENCES

Appl, S., Camenzind, M., 1988, A&A, 206, 258
Ballard K.R., Heavens A.F., 1991, MNRAS, 251, 438
Ballard K.R., Heavens A.F., 1992, MNRAS, 259, 89
Begelman M.C., Kirk J.G., 1990, ApJ, 353, 66
Drolias B., Quenby J.J., 1994, preprint
Ellison D.C., Jones F.C., Reynolds S.P., 1990, ApJ, 360, 702
Heavens A., Drury L'O.C., 1988, MNRAS, 235, 997
Jones F.C., Ellison D.C., 1991, Space Sci. Rev., 58, 259
Kirk J.G., 1988, *Habilitation Theses*, preprint No. 345, Max-Planck-Institut für Astrophysik, Garching
Kirk J.G., Heavens A., 1989, MNRAS, 239, 995
Kirk J.G., Schneider P., 1987a, ApJ, 315, 425
Kirk J.G., Schneider P., 1987b, ApJ, 322, 256
Kirk J.G., Schneider P., 1988, A&A, 201, 177
Lagage P.O., Cesarsky C., 1983, A&A, 125, 249
Lieu, R., Quenby, J.J., Drolias, B., Naidu, K., 1994, ApJ, 421, 211
Lucek, S.G., Bell, A.R., 1994, MNRAS, 268, 581
Naito T., Takahara F., 1995, MNRAS, 275, 1077
Newman P.L., Moussas X., Quenby J.J., Valdes-Galicia J.F., Theodossiou-Ekaterinidi Z., 1992, A&A, 255, 443
Ostrowski M., 1988, MNRAS, 233, 257
Ostrowski M., 1991, MNRAS, 249, 551
Ostrowski M., 1993, MNRAS, 264, 248
Ostrowski M., 1994a, Comments on Astrophysics, 17, 207
Ostrowski M., 1994b, in Proc. 26th Meeting of the Polish Astr. Soc., eds. M. Sarna & J. Zalewski, Warszawa
Ostrowski M., Bednarz, J., 1995, in Proc. 24th Int. Cosmic Ray Conf., 3, 257, Rome.
Ostrowski M., Schlickeiser R., 1996, Solar Physics (in press; see, also in Proc. 24th Cosmic Ray Conf. 3, 297, Rome)
Quenby J.J., Lieu R., 1989, Nature 342, 654.
Quenby J.J., Drolias B., 1995, in Proc. 24th Int. Cosmic Ray Conf., 3, 261, Rome.
Takahara F., Terasawa T., 1990, in Proc. ICRR Int. Symp. on "Astrophysical Aspects of the Most Energetic Cosmic Rays", Kofu.